Indian Statistical Institute,Bangalore Centre

B.Math.(Hons.)II Year-2013-14,First Semester

Optimization

Mid-Term Exam

16 Sept 2013, 10am - 1pm.

Instructor: P.S.Datti

Max.Marks: 30

NOTE: Solve all the questions. All questions carry equal marks.WRITE NEATLY.

1. Determine all the projections $P \in \mathbb{R}^{2\times 2}$ and identify the orthogonal projections among them.

2. Suppose $A = ((a_{ij}))$ is the $n \times n$ matrix given by

$$a_{ij} = \begin{cases} 2 & \text{if } i = j \\ -1 & \text{if } |i-j| = 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show by direct computation that $(Ax, x) > 0$ for all non-zero $x \in \mathbb{R}^n$.

   (b) Now let $n = 3$. Solve $Ax = b$, using a method of your choice, where

$$b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

3. (a) Given that the 2×2 real matrix

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

   is of rank 1, derive the formula $A^{+} = (a^2 + b^2 + c^2 + d^2)^{-1} A^t$ for the Moore-Penrose(i.e.,the generalized) inverse of A.

   (b) Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{pmatrix}, \quad b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.$$

   Find the least square solution of min $\{||Ax - b|| : x \in \mathbb{R}^3\}$.

4. Let $A \in \mathbb{R}^{2\times 2}$ and consider the symmetric matrix

$$B = \begin{pmatrix} O & A \\ A^t & O \end{pmatrix},$$

where O denotes the 2×2 zero matrix. Arrange the eigenvalues of B as $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$. If the singular values of A are arranged as $\mu_1 \geq \mu_2$, express $\lambda_i, i = 1, 2, 3, 4$ in terms of $\mu_i, i = 1, 2$.

5. For what values of the real numbers λ, μ is the $n \times n$ matrix $I + \lambda E_{1n} + \mu E_{n1}$ non-singular? In that case find its inverse.